Trajectory Adaptation from Demonstrations with Constrained Optimization

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Abstract— **This paper proposes an approach for the adaptation of robot trajectories taken from a set of demonstrations. The problem is formulated as a constrained optimization problem where the set of demonstrations are used as target values to build a Quadratic Program (QP). The constraints constitute the adaptation's conditions of the new trajectory, e.g. new initial or final points or keep the trajectory within a specific range. The performance of our approach is verified in the adaptation of a set of demonstrations taken from a Panda robot for new conditions.**

I. INTRODUCTION

Imitation learning approaches aim to generalize tasks to novel situations. Most approaches are designed under a learning framework, where a given criteria is minimized. A variety of approaches exist where a set of task demonstrations are used to train a given model to provide generalization for new different conditions. Some representative approaches within this field are i.g. Task-Parameterized Gaussian Mixture Model (TP-GMM) [2] that considers as task parameters, the homogeneous transformations between arbitrary reference frames. By observing human demonstrations from each of these frames the robot is able to learn the spatial relationship between start, goal, and via points in the trajectory. Conditional Neural Movement Primitives (CNMPs) Seker et al. [3] generate motion trajectories by sampling observations from the training data and predicting a conditional distribution over target points, e.g. robot position, forces, and any task parameters. However, CNMPs have limited extrapolation capabilities. A possibility to improve the extrapolation performance is to combine imitation and reinforcement learning [1]. In order to maximize the generalization to new conditions, these models require a training process aimed to maximize the adaptation capabilities by minimizing a given loss function. However, there is no way to guarantee the conditions will be fully reached for the adaptation. Besides, the nature of some manipulation tasks requires reaching a certain level of precision for the new conditions to be adapted to. In this paper, we tackled the adaptation problem by a constrained optimization approach that uses a set of demonstrations as target points to build a linear regression model using a set of Basis Functions (BF). The conditions to meet by the adaptation are defined as constraints of the QP. In this way, it is possible to satisfy the new conditions that requires the adaptation. Our approach allows defining equality and inequality constraints at both position and velocity levels. Our approach is validated in the adaptation of a set of demonstrations taken from a Panda robot, where the adaptations involve different equality and inequality constraints at the same time.

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II. METHOD

Given a set of N observations $D = [\{t_1, \mathbf{y}_1\}, \cdots, \{t_N, \mathbf{y}_N\}]$ where $t \in \mathbb{R}$ defines the independent variable and $\mathbf{y} \in \mathbb{R}^d$ the target values of dimension d. The goal is to find a set of parameters $\mathbf{w} \in \mathbb{R}^M$ that minimize the sum of squared errors:

$$E_D(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}(\mathbf{y}(t_n, \mathbf{w}) - \mathbf{y}_n)^2 \quad (1)$$

The model can be defined as a linear combination of fixed, nonlinear BF $\phi(t)$, i.e., $\mathbf{y}(t, \mathbf{w}) = w_0 + \sum_{j=1}^{M-1} w_j\phi_j(t) = \mathbf{w}^{\mathrm{T}}\phi(t)$, where $M-1$ is the number of BF.

The regression problem can be rewritten as a QP that allows to impose constraints, in the form

$$\begin{aligned} \underset{\mathbf{w}^*}{\text{minimize}} \quad & \tfrac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{P}\mathbf{w} + \mathbf{q}^{\mathrm{T}}\mathbf{w} \\ \text{s.t.} \quad & \begin{cases} \mathbf{y}_l \le \mathbf{G}\mathbf{w} \le \mathbf{y}_u \\ \mathbf{A}\mathbf{w} = \mathbf{y}_A \\ \dot{\mathbf{y}}_l \le \dot{\mathbf{G}}\mathbf{w} \le \dot{\mathbf{y}}_u \end{cases} \end{aligned} \quad (2)$$

where $\mathbf{w}^*$ is the optimal vector that minimize the Sum of squared errors (SSE) given in Eq. 1; $\mathbf{P} = 2\mathbf{M}^{\mathrm{T}}\mathbf{M}$, and $\mathbf{q} = -2\mathbf{M}^{\mathrm{T}}\mathbf{c}$ defines the standard form expressions, $\mathbf{M} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}$ and $\mathbf{c} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{y}$; where $\mathbf{y} \in \mathbb{R}^{dN}$ is the stacked vector of target values and $\mathbf{\Phi} \in \mathbb{R}^{N \times M}$ is know as the *design matrix*

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \phi_1(t_1) & \ldots & \phi_{M-1}(t_1) \\ 1 & \phi_1(t_2) & \ldots & \phi_{M-1}(t_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \phi_1(t_N) & \ldots & \phi_{M-1}(t_N) \end{bmatrix}. \quad (3)$$

$\mathbf{A}\mathbf{w} = \mathbf{y}_A$ defines the equality constraints at the position level constructed from a predefined set of P data points $D^A = \{\mathbf{t}_A, \mathbf{y}_A\}$, where $\mathbf{y}_A \in \mathbb{R}^{P \times d}$ defines the desired values of the regression evaluated at $\mathbf{t}_A \in \mathbb{R}^P$. The matrix $\mathbf{A} \in \mathbb{R}^{P \times M}$ is calculated as $\mathbf{A} = \mathbf{\Phi}(\mathbf{t}_A)$. On the other hand, $\mathbf{y}_l \le \mathbf{G}\mathbf{w} \le \mathbf{y}_u$ represent the inequality constraints at the position level and is constructed from a set of Q data points $D^G = \{\mathbf{t}_G, \mathbf{y}_l, \mathbf{y}_u\}$, where $\mathbf{y}_l, \mathbf{y}_u \in \mathbb{R}^{Q \times d}$ are the lower and upper boundaries data points respectively and the matrix $\mathbf{G} \in \mathbb{R}^{Q \times M}$ is calculated as $\mathbf{G} = \mathbf{\Phi}(\mathbf{t}_G)$. The inequality constraints are used to keep the regression values evaluated at $\mathbf{t}_G \in \mathbb{R}^Q$ within the range $[\mathbf{y}_l, \mathbf{y}_u]$. Finally, $\dot{\mathbf{y}}_l \le \dot{\mathbf{G}}\mathbf{w} \le \dot{\mathbf{y}}_u$ represents the inequality constraints at velocity level that are defined from a set of

V datapoints $\dot{D}^G = \{\mathbf{t}_v, \dot{\mathbf{y}}_l, \dot{\mathbf{y}}_u\}$, where $\dot{\mathbf{y}}_l, \dot{\mathbf{y}}_u \in \mathbb{R}^{V \times d}$ are the lower and upper velocity boundaries datapoints respectively and the matrix $\dot{\mathbf{G}}$ is calculated as $\dot{\mathbf{G}} = \dot{\mathbf{\Phi}}(\mathbf{t}_v)$, where $\dot{\mathbf{\Phi}} \in \mathbb{R}^{V \times M}$ defines the partial derivatives of the BF $\dot{\mathbf{\Phi}} = \frac{\partial \phi(x)}{\partial x}$. The velocity inequality constraints are used to keep the velocity of the regression evaluated at $\mathbf{t}_v$ within the range $[\dot{\mathbf{y}}_l, \dot{\mathbf{y}}_u]$. This is specially useful to generate smooth trajectory motions in the reproduction of the adapted task. By solving the QP in Eq. 2, it is possible to find the optimal vector $\mathbf{w}^*$ that minimizes the sum of squares errors (1) and satisfies at the same time the equality and inequality constraints (2).

III. Results

This section presents the results obtained from applying our method to robot trajectory adaptation. For this experiment, we have used a dataset of 9 different $2D$ trajectories with 700 data points each, forming a dataset $D = \{\mathbf{t}, \mathbf{y}\}$ where $\mathbf{t} \in \mathbb{R}^{2700}$, with values within the range $[0, 1]$; whereas the target values are $\mathbf{y} \in \mathbb{R}^{2700 \times 2}$. The used Basis Functions is conformed by a set of 18 functions $\phi(t) = [1, t, \sin(\alpha_0 t), \cos(\alpha_0 t), \cdots, sin(\alpha_7 t), \cos(\alpha_7 t)]$ with $\alpha_i \in \{0.1, 1, 5, 10, 20, 30, 40, 50\}$. The set of BF and their parameters were selected empirically motivated from the Fourier BF.

The first experiment is shown in the Fig. 1 Case I. The adaptation includes new initial and final points of the trajectory which are defined as two equality constraints for $t = 0$ and $t = 1$, indicated for purple markers. We have also defined a set of inequality constraints in Y axis as $\{\mathbf{t}, \mathbf{y}_u = 0.055\}$, (blue light area), used to keep the Y axis trajectory values lower than $\mathbf{y}_u$. The plot presents the results for three different tuples of initial and final points. In these results, the adapted trajectory fully satisfy the new initial and final conditions as well as the imposed inequality constraint in the Y axis, and most importantly, keeping the shape of the trajectory, which means, the new obtained trajectory has a similar shape that the demonstrations.

In Fig. 1 Case II a second adaptation case is shown using the same data set. Here, we present a comparison between two adapted trajectories, the orange one is adapted only in position and the blue one is adapted in position and velocity. The conditions of adapted position for both trajectories are the same, indicated by the purple markers. For the blue adapted trajectory, the velocity constraint is defined within the range $[-0.55, 0.55]$. The velocity for both trajectories is shown in Fig. 1c). The orange trajectory moves freely due to the lack of constraint, whereas the blue trajectory remains within the imposed velocity range defined in the inequality constraint. In Fig. 1b) the respective position trajectories are shown. Both trajectories satisfied the initial and final adaptation conditions and both keep the shape of the trajectory overall. However, the blue trajectory will be the one that produces smoother motions in the reproductions due to the velocity constraints.

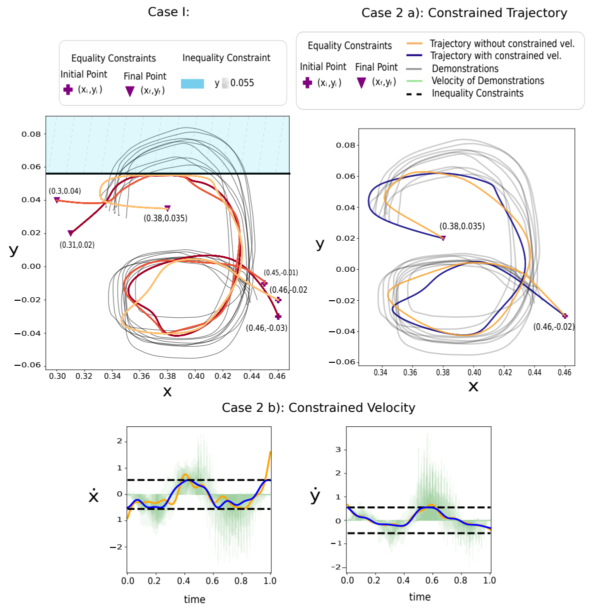

Fig. 1. Case I: Adaptation for new initial and final positions with a constraint in the Y axis. Case II: Adaptation for a new initial and final position with velocity constraints.

IV. Conclusions

The proposed approach addresses the adaptation problem for new conditions at both position and velocity level for a set of demonstrations. The approach is defined as a regression problem and handled as Constrained Quadratic Optimization, where the criteria to be minimized is defined by a sum-of-square errors of data points of the demonstrations, and the constraints represent the new adaptation conditions. The approach is validated in a set of trajectories taken from a Panda Robot. The adaptation involves new initial and final points as well as velocity constraints. The results show our proposed approach can fully satisfy the new imposed adaptation conditions while keeping the shape of the trajectory overall. The approach has important relevance 1) to scenarios with continuous changes that demand continuous adaptations of the trajectory, 2) to adaptations that require the shape of the trajectory to be preserved, and 3) to trajectories that demands high level of accuracy for the new adapted conditions. Our approach considers the following future work: I) Introduction of slack variables in the optimal solution vector, which are essential to relax the constraints and guarantee feasible solutions of the QP. II) Extend the adaptation to 3D trajectories and reproduce them in real scenarios. III) Comparison with similar methods e.g. CNMP, TP-GMM. IV) Explore some methods for better selection of the BF.

References

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